GENERAL INVESTIGATIONS ON THE MORTALITY AND THE MULTIPLICATION OF MAN

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1. The registers of births and of deaths at each age, which are published in several places every year, furnish so many different questions about the mortality and multiplication of man, that it would take too long to relate them all. Yet, some of them depend for the most part on others, so that having developed one or two of them, all the others are likewise settled. Since the solutions must be drawn from the registers mentioned, it should be noted that these registers differ greatly, according to the diversity of towns, villages and provinces where they were drawn up; and for the same reason, the solutions to all these questions depend greatly on the registers on which they are based. That is why I intend to discuss most of these questions in general here, without limiting myself to the results that the registers from a particular place furnish; then it will be easy to make the application to whichever places we wish.

2. In fact, I observe first that these questions taken in general depend on two hypotheses, which being clearly established, it becomes easy to derive from them the solutions to all. I will call the first the *hypothesis of mortality*, by which we determine how many of a certain number of men, who are born at the same time, will still be alive after each passing year. Here considerations of multiplication do not count at all. And so it is necessary to constitute the second hypothesis, which I will call the *hypothesis of multiplication*, by which I indicate how much the total number of all men is augmented or diminished during the course of a year. This hypothesis depends, then, on the number of marriages and on fertility, while the first is based on the vitality which is characteristic of the men.

I. Hypothesis of Mortality

3. For the first hypothesis, let us imagine an arbitrary number N of infants who are born at the same time, and I will indicate the number of those who are still alive at the end of one year by $(1)N$, those who are still alive at the end of two years by $(2)N$, of three years by $(3)N$, of four by $(4)N$, and

Recherches générales sur la mortalité et la multiplication du genre humain, Mémoires de l'académie des sciences de Berlin **16** (1767), 144–164. Number 334 in the Eneström index.

so on. This is the notation I use to indicate how the number of men born at the same time decreases successively, which will have for each region and each manner of living particular values. Nevertheless, we can note that the numbers indicated by

$$(1),\ (2),\ (3),\ (4),\ (5),\ \text{etc.}$$

form a decreasing progression of fractions, of which the largest, (1), is less than unity, and when we continue these terms beyond 100, they so strongly decrease that they vanish almost entirely. For, if from 100 million men none attain the age of 125 years, it is necessary that the term (125) be less than 1/100,000,000.

4. Having established for a certain place by a big enough number of observations the values of the fractions (1), (2), (3), (4), etc., we can solve a number of questions that are ordinarily asked about the probability of human life. First, it is clear that if the number of infants born at the same time is N, then according to probability, they will die throughout the years as indicated in this table:

From year	*to year*	*this many will die*
0	1	$N - (1)N$
1	2	$(1)N - (2)N$
2	3	$(2)N - (3)N$
3	4	$(3)N - (4)N$
4	5	$(4)N - (5)N$
	etc.	

Just as from this number N there will probably be $(n)N$ living at the end of n years, so it follows that the number of deaths during this term of n years will be $N - (n)N$. After this remark, I will give the solution to the following questions.

5. *Given a certain number of people who are all the same age, determine how many will probably still be alive after a specified number of years.*

Suppose there are M people who are each m years old, and we ask how many of them will probably still be alive after n years. We have $M = (m)N$ and so $N = M/(m)$, where N indicates the number of all the infants born at the same time, of which M remain alive after m years. Now of this same number, $(m+n)N$ will probably still be alive after $m+n$ years since their birth, and therefore after n years since the time proposed. So the number sought in the question is

$$\frac{(m+n)}{(m)}M$$

or after n years there will probably still be that many living among the M people who are presently m years old.

So it is probable that of the M men of age m,

$$\left[1-\frac{(m+n)}{(m)}\right]M$$

of them will die before n years pass.

6. *Find the probability that a man of a certain age will still be alive after a certain number of years.*

Let the age of the man in question be m years, and we seek the probability that this man will still be alive at the end of n years. We imagine M men of the same age, and since, after n years, there will probably be

$$\frac{(m+n)}{(m)}M$$

of them still alive, the probability that the given man will be among this number is

$$\frac{(m+n)}{(m)}$$

So the probability that this man will die before the end of these n years is

$$1-\frac{(m+n)}{(m)}$$

and therefore the hope that this man will not die in the next $m+n$ years is to the fear of his dying in this same interval as $(m+n)$ to $(m)-(m+n)$. So the hope will surpass the fear if $(m+n) > \frac{1}{2}(m)$; and the fear will be better founded if $(m+n) < \frac{1}{2}(m)$; and the fear equal to the hope if $(m+n) = \frac{1}{2}(m)$.

7. *We ask the probability that a man of a certain age will die in a given year.*

Suppose that the man in question is m years old, and we ask the probability that he will attain the age of n years, but die before he reaches age $n+1$. To find this probability, we imagine a large number, M, of men of the same age, and having $M = (m)N$ and $N = M/(m)$, there will be $\frac{(n)}{(m)}M$ men who attain the age of n years, and $\frac{(n+1)}{(m)}M$ who attain that of $n+1$ years. Therefore during the course of this year, there will probably die

$$\frac{(n)-(n+1)}{(m)}M$$

men, and so the probability that the given man will find himself among that number will be

$$\frac{(n)-(n+1)}{(m)}$$

From this it is clear that in order for this same man to die between year n and year $n+v$, the probability will be

$$\frac{(n)-(n+v)}{(m)}$$

Now, for this man to die on a given day of the specified year, the probability will be

$$\frac{(n)-(n+1)}{365(m)}$$

If the question is of a newborn infant, we have only to write 1 in place of the fraction (m).

8. *Find the term that a man of a given age can hope to reach, so that it is equally probable that he dies before the end of this term as after.*

Let the age of the man in question be m years, and that which he can hope to attain z years, which is necessary to find. Now, the probability that he reaches this age is $(z)/(m)$, so the probability that he dies before the end of this term is $1-(z)/(m)$. Therefore, since the one probability must be the same as the other, we have this equation

$$\frac{(z)}{(m)} = 1 - \frac{(z)}{(m)}$$

and therefore

$$(z) = \frac{1}{2}(m)$$

from which it is easy to find the number z, as soon as we have determined by observation the values of all the fractions

$$(1),\ (2),\ (3),\ (4),\ (5),\ (6),\ \text{etc.}$$

because we will see immediately which (z) is half of the given (m).

Having found this number z, we call the interval $z - m$ the *life force* of a man of m years.

9. *Determine the life annuity that it is fair to pay to men of an arbitrary age each year until their deaths, for a sum paid in advance.*

We imagine M men, all of the same age m, and each pays in advance the sum a, which furnishes a fund equal to Ma. Let x be the sum that we must pay to each every year during his life. After one year, the fund must pay

$$\frac{(m+1)}{(m)} Mx$$

after two years

$$\frac{(m+2)}{(m)} Mx$$

after three

$$\frac{(m+3)}{(m)} Mx$$

and so on.

Now, assuming that the fund is invested at 5 percent, a sum S payable after n years is worth only $\left[\frac{20}{21}\right]^n S$ now. However, to make our result more general, we suppose that a sum S grows with interest in one year to λS, and $1/\lambda$ will be what we indicated by $\frac{20}{21}$. But a sum S payable at the end of n years will be worth only S/λ^n now. From that we set up the following calculation

After	*payment due*	*present value*
1 year	$\frac{(m+1)}{(m)} Mx$	$\frac{(m+1)}{(m)} \cdot \frac{Mx}{\lambda}$
2 years	$\frac{(m+2)}{(m)} Mx$	$\frac{(m+2)}{(m)} \cdot \frac{Mx}{\lambda^2}$
3 years	$\frac{(m+3)}{(m)} Mx$	$\frac{(m+3)}{(m)} \cdot \frac{Mx}{\lambda^3}$

etc.

Now equity requires that all these sums, reduced to the present time, be equal to the entire fund, Ma, from which we draw this equation

$$a = \frac{x}{(m)}\left[\frac{(m+1)}{\lambda} + \frac{(m+2)}{\lambda^2} + \frac{(m+3)}{\lambda^3} + \frac{(m+4)}{\lambda^4} + \text{etc.}\right]$$

and therefore the amount that the fund must pay yearly to each of the parties is

$$x = \frac{(m)a}{\frac{(m+1)}{\lambda} + \frac{(m+2)}{\lambda^2} + \frac{(m+3)}{\lambda^3} + \frac{(m+4)}{\lambda^4} + \text{etc.}}$$

Knowing then the values of all these fractions (1), (2), (3), etc., it is easy to find the sum x that is owed to each man aged m years, returned by a given rate of interest.

10. *When the parties are newborn infants and the life annuity payments do not become due until they attain a certain age, determine the amount of these payments.*

Suppose that we pay the sum a for each newborn infant and that he is due to receive payments only when he attains the age of n years, and starting then, we pay him each year the sum x, which must be determined. Counting then the interest as before, we arrive at this equation

$$a = x\left[\frac{(n)}{\lambda^n} + \frac{(n+1)}{\lambda^{n+1}} + \frac{(n+2)}{\lambda^{n+2}} + \frac{(n+3)}{\lambda^{n+3}} + \text{etc.}\right]$$

which yields

$$x = \frac{a}{\frac{(n)}{\lambda^n} + \frac{(n+1)}{\lambda^{n+1}} + \frac{(n+2)}{\lambda^{n+2}} + \frac{(n+3)}{\lambda^{n+3}} + \text{etc.}}$$

From this it is clear that such an annuity can become very advantageous, and that a man, when he attains a certain age, can enjoy a considerable income all his life, at little cost.

11. All these questions, then, are easily solved once we know the values of the fractions (1), (2), (3), (4), etc., which depend both on the region and on the manner of living. Also, it has been noted that these values are different for the two sexes, so that they cannot be determined generally. Now, in order to infer them from observations, one easily understands, it is necessary to have a large number of them, which furthermore extends over all types of people; and in this regard one cannot use the registers of life annuities, which start with infants under one year. Because first of all, we cannot regard these infants as newly born, and most have undoubtedly already escaped the dangers of the first months. Second, one will hardly ever take on infants of a feeble constitution, so that we must regard as select the infants for which one takes a life annuity. Thus the values of our fractions (1), (2), (3), etc. that one infers from the registers of life annuities will be unfailingly too large, especially with regard to the first years. However, since it is necessary to adjust the annuities from such registers rather than from true mortality, I will add here the values of our fractions, just as they are drawn from the observations of Mr. Kersseboom.

(1) = 0.804	(25) = 0.552	(49) = 0.370	(73) = 0.145
(2) = 0.768	(26) = 0.544	(50) = 0.362	(74) = 0.135
(3) = 0.736	(27) = 0.535	(51) = 0.354	(75) = 0.125
(4) = 0.709	(28) = 0.525	(52) = 0.345	(76) = 0.114
(5) = 0.688	(29) = 0.516	(53) = 0.336	(77) = 0.104
(6) = 0.676	(30) = 0.507	(54) = 0.327	(78) = 0.093
(7) = 0.664	(31) = 0.499	(55) = 0.319	(79) = 0.082
(8) = 0.653	(32) = 0.490	(56) = 0.310	(80) = 0.072
(9) = 0.646	(33) = 0.482	(57) = 0.301	(81) = 0.063
(10) = 0.639	(34) = 0.475	(58) = 0.291	(82) = 0.054
(11) = 0.633	(35) = 0.468	(59) = 0.282	(83) = 0.046
(12) = 0.627	(36) = 0.461	(60) = 0.273	(84) = 0.039
(13) = 0.621	(37) = 0.454	(61) = 0.264	(85) = 0.032
(14) = 0.616	(38) = 0.446	(62) = 0.254	(86) = 0.026
(15) = 0.611	(39) = 0.439	(63) = 0.245	(87) = 0.020
(16) = 0.606	(40) = 0.432	(64) = 0.235	(88) = 0.015
(17) = 0.601	(41) = 0.426	(65) = 0.225	(89) = 0.011
(18) = 0.596	(42) = 0.420	(66) = 0.215	(90) = 0.008
(19) = 0.590	(43) = 0.413	(67) = 0.205	(91) = 0.006
(20) = 0.584	(44) = 0.406	(68) = 0.195	(92) = 0.004
(21) = 0.577	(45) = 0.400	(69) = 0.185	(93) = 0.003
(22) = 0.571	(46) = 0.393	(70) = 0.175	(94) = 0.002
(23) = 0.565	(47) = 0.386	(71) = 0.165	(95) = 0.001
(24) = 0.559	(48) = 0.378	(72) = 0.155	

Now, since this table is drawn up based on select infants who have indeed already survived some months since their births, if one wants to apply it to all the newborn infants in a town or province, it is necessary to decrease all these numbers by a certain amount, to take into account the large mortality to which the infants are subject immediately after their births. But we will draw this correction more certainly from observations involving multiplication, which I will consider now.

II. Hypothesis of Multiplication

12. This hypothesis is based on the principle of propagation, from which it is immediately obvious that if each year just as many infants are born as there are men who die, the number of all men will stay the same, and there will not be any multiplication whatsoever. But if the number of infants born each year surpasses the number of deaths, each year will produce an increase in the number of living, which will equal the excess of births over deaths. Now this increase will change to a decrease when the number of deaths surpasses that of births. From this we have three cases to consider: the first where the number of men stays constantly the same; the second where it increases each year, and the third where it decreases each year. So if M indicates the number of all the men living at present and mM the number of those who are living next year, the first case happens if $m = 1$, the second if $m > 1$, and the third if $m < 1$; so that all the cases can be included in the general coefficient m.

13. Now, having fixed the principle of propagation, which depends on marriages and on fertility, it is clear that the number of infants born during the course of a year must stand in a certain ratio to the number of all the living men. From this it follows that if the number of living stays the same, each year there will be born the same number of infants; and if the number of living increases or decreases, the number of births must increase or decrease in the same ratio. So, in comparing together the number of all the births during several consecutive years, according as this number stays the same, or increases, or decreases, one can infer from it whether the number of all men stays the same, or increases, or decreases. By applying the principle of mortality, it is also clear that the number of deaths during a year must stand in a certain ratio to both the number of all the living and also to that of all the births.

14. Since these two principles of mortality and propagation are independent of one another, and as I considered the first independently of the other, we can also present the latter without reference to the first. Because, supposing the number of all the living at the same time is M, the number of infants who are produced by them in the space of a year can be set to αM, so that α is the measure of the propagation or of the fertility. But it is difficult to draw from this position the consequences regarding multiplication and the other phenomena that depend on them. The reasoning will become more clear if we introduce, at the beginning of the calculation, the number of infants born each year, and then apply the hypothesis of mortality. Then we can infer the value of α. Conversely then, the number of births depends at once on the two hypotheses of mortality and fertility; and from there, one will then draw without difficulty the solution to all the other questions that are ordinarily asked in treating this matter.

15. Just as I suppose that the rule of mortality always stays the same, I will suppose a similar constance of fertility, so that the number of infants born each year will always be proportional to the number of all the living. So, if the number of the living stays the same, we will also have each year the same number of births; and if the number of the living increases or decreases, the number of annual births will increase or decrease by the same ratio.

Let N then be the number of infants born during the course of a year, and nN that of the infants born the next year, and since the ratio which changed N to nN persists afterwards, it is necessary that from any year whatsoever to the next the number of births increases by the ratio of 1 to n. Consequently, the third year will have n^2N births, the forth n^3N, the fifth n^4N, and so on. Indeed, the numbers of annual births constitute a geometric progression, either increasing, decreasing, or remaining the same, according as $n > 1$, $n < 1$, or $n = 1$.

16. Let us suppose, then, that in a town or province, the number of infants born this year is N, and the number born next year will be nN, and so on, according to this progression

	Number of births
at present	N
after 1 year	$n\ N$
after 2 years	n^2N
after 3 years	n^3N
after 4 years	n^4N

etc.

and if we suppose that after 100 years nobody alive at present will still be alive, then there will not be, after 100 years, anybody alive other than those still alive from these births. Then, applying the hypothesis of mortality, we will be able to determine the number of all the men alive after 100 years. Now, since there will be born that year $n^{100}N$, we will have the quotient of the births and the number of all the living.

17. To make this more clear, let us see how many men will still be alive after 100 years, from births during each preceding year.

	Number of births	*after 100 years still alive*
at present	N	$(100)\ N$
after 1 year	nN	$(99)n\ N$
after 2 years	n^2N	$(98)n^2N$
after 3 years	n^3N	$(97)n^3N$
⋮	⋮	⋮
after 98 years	$n^{98}N$	$(2)n^{98}N$
after 99 years	$n^{99}N$	$(1)n^{99}N$
after 100 years	$n^{100}N$	$n^{100}N$

Therefore, the number of all the living after 100 years will be

$$n^{100}N\left[1+\frac{(1)}{n}+\frac{(2)}{n^2}+\frac{(3)}{n^3}+\frac{(4)}{n^4}+\frac{(5)}{n^5}+\text{etc.}\right].$$

18. The terms of this series will eventually vanish by virtue of the hypothesis of mortality; and since the number of all the living stands in a certain ratio to the number of births during the course of a year, the multiplication from one year to the next, which we suppose to be 1 to n, reveals this ratio to us. Because, if the number of all the living is M and the number of infants who are procreated by them during the course of a year is set to N, we will have

$$\frac{M}{N}=1+\frac{(1)}{n}+\frac{(2)}{n^2}+\frac{(3)}{n^3}+\frac{(4)}{n^4}+\frac{(5)}{n^5}+\text{etc.}$$

So, if we know the quotient M/N and if we apply the hypothesis of mortality, or the values of the fractions (1), (2), (3), (4), etc., this equation determines conversely the ratio of multiplication, $1 : n$, from one year to the next. However, one clearly sees that this determination cannot be developed in general, but, for each hypothesis of mortality, if one calculates the quotient M/N for several values of n and if from them we draw up a table, it will be easy to conversely assign for each given quotient, $M : N$, which expresses fertility, the annual growth of all the living, which is the same as that of births.

19. Suppose then that the hypothesis of mortality or the fractions

$$(1),\ (2),\ (3),\ (4),\ (5),\ \text{etc.}$$

are known, as well as the hypothesis of fertility or the quotient of all the living, M, and the number of infants, N, who are procreated by them during a year. We will recognize from this whether the number of men is staying invariable, or if it is increasing, or decreasing. Because, if we set the number of all the living next year to nM, those living at present being M, it is necessary to draw the value of n from the previously-found equation

$$\frac{M}{N} = 1 + \frac{(1)}{n} + \frac{(2)}{n^2} + \frac{(3)}{n^3} + \frac{(4)}{n^4} + \frac{(5)}{n^5} + \text{etc.}$$

and supposing the resolution of this equation is known, it is immaterial whether one knows the fertility, M/N, or the multiplication $1 : n$, the one being determined by the other, by means of the hypothesis of mortality.

20. *The hypotheses of mortality and fertility being given, if one knows the number of all the living, find how many of them there will be of each age.*

Let M be the number of all the living, and N the number of infants who are procreated by them in a year. By the hypothesis of mortality, we will know the ratio of the annual multiplication, $1 : n$. Then, knowing the value of n, it is easy to infer from §17 that there will be, among the number M,

N infants newly born,

$\frac{(1)}{n}N$ aged one year,

$\frac{(2)}{n^2}N$ aged two years ,

$\frac{(3)}{n^3}N$ aged three years,

$\frac{(4)}{n^4}N$ aged four years,

and in general

$\frac{(a)}{n^a}N$ aged a years.

Indeed, the sum of all these numbers taken together is M.

21. *The same things being given, find the number of men who will die each year.*

Let M be the number of men living at present, including the infants born this year, of which the number is N. The quotient M/N will determine the annual increase, which is $1 : n$. Then the next year the number of living will be nM, among which the number of newborns will be nN. The others, of which the number is $nM - nN$, are those who are still alive from the preceding year, of which the number was M. From that it follows that the number of them that will die is

$$[1 - n]M + nN.$$

So if the number of the living is M, there will die during the course of a year $[1 - n]M + nN$, whereas in this same time there will be born N infants.

22. *Knowing both the number of births and the number of burials that happen during the course of a year, find the number of all the living and their annual increase, for a given hypothesis of mortality.*

Let N be the number of births and O the number of burials that happen in a year; then, set the number of all the living to M and the annual augmentation to $1 : n$, and the preceding solution furnishes us this equation

$$O = [1 - n]M + nN$$

Now, the hypothesis of mortality gives

$$\frac{M}{N} = 1 + \frac{(1)}{n} + \frac{(2)}{n^2} + \frac{(3)}{n^3} + \frac{(4)}{n^4} + \text{etc.}$$

Therefore, having by the first

$$M = \frac{O - nN}{1 - n}$$

this value, when substituted in the other equation, gives

$$\frac{1}{N} \cdot \frac{O - N}{1 - n} = \frac{1}{N} \cdot \frac{N - O}{n - 1} = \frac{(1)}{n} + \frac{(2)}{n^2} + \frac{(3)}{n^3} + \text{etc.}$$

from which it is necessary to find the value of the number n.

23. If the number of burials, O, is equal to that of the births, N, so that

$$N = [1 - n]M + nN$$

it is absolutely necessary that $n = 1$, or that the number of living stay the same; and in this case, this number will be

$$M = N[1 + (1) + (2) + (3) + (4) + \text{etc.}]$$

Now, if the number of births, N, surpasses that of the burials, O, so that $N - O$ is a positive number, the equation

$$\frac{1}{N} \cdot \frac{N - O}{n - 1} = \frac{(1)}{n} + \frac{(2)}{n^2} + \frac{(3)}{n^3} + \frac{(4)}{n^4} + \text{etc.}$$

will give for n a value greater than 1, which indicates that the number of the living is increasing. But, if the number of births, N is smaller than that of the burials, O, our equation must be represented in this form

$$\frac{1}{N} \cdot \frac{O - N}{1 - n} = \frac{(1)}{n} + \frac{(2)}{n^2} + \frac{(3)}{n^3} + \frac{(4)}{n^4} + \text{etc.}$$

from which we draw for n a value smaller than 1, which indicates that the number of living is decreasing.

24. *The number of births and of burials in a year being given, find how many of each age there will be among the dead.*

Let N be the number of infants born during a year and O the number of deaths. By the preceding question we will have the number of all the living, M, with the multiplication $1 : n$ from one year to the next. From there, let us consider how many men will be alive at each age, both this year and the next.

Number	*this year*	*next year*
of newly born	N	nN
of age 1 year	$\frac{(1)}{n}N$	$(1)N$
of age 2 years	$\frac{(2)}{n^2}N$	$\frac{(2)}{n}N$
of age 3 years	$\frac{(3)}{n^3}N$	$\frac{(3)}{n^2}N$
	etc.	

From which it is clear that the following number of them will die during the course of this year:

	Number of deaths
under 1 year	$[1-(1)]\,N$
from 1 to 2 years	$[(1)-(2)]\,\frac{N}{n}$
from 2 to 3 years	$[(2)-(3)]\,\frac{N}{n^2}$
from 3 to 4 years	$[(3)-(4)]\,\frac{N}{n^3}$
from 4 to 5 years	$[(4)-(5)]\,\frac{N}{n^4}$

etc.

25. The number of all the dead this year being O, we have this equation

$$\frac{O}{N} = 1-(1)\left[1-\frac{1}{n}\right]-\frac{(2)}{n}\left[1-\frac{1}{n}\right]-\frac{(3)}{n^2}\left[1-\frac{1}{n}\right]-\text{etc.}$$

which agrees with

$$O = [1-n]\,M + nM$$

because of

$$\frac{M}{N} = 1+\frac{(1)}{n}+\frac{(2)}{n^2}+\frac{(3)}{n^3}+\frac{(4)}{n^4}+\frac{(5)}{n^5}+\text{etc.}$$

So, knowing the hypothesis of mortality along with with the annual multiplication $1:n$, and the number N of births from one year, we can determine how many men of each age will probably die during the course of a year.

26. *Knowing the number of all the living, as well as the number of births with the number of deaths at each age during the course of a year, find the law of mortality.*

Let M be the number of all the living, N that of the births, and O of burials during the course of a year; and from that we will immediately know the annual multiplication

$$n = \frac{M-O}{M-N}$$

Then for this year, the number of deaths will be, by the preceding question

under 1 year	$\alpha = [1 - (1)]\,N$
from 1 year to 2 years	$\beta = [(1) - (2)]\,\dfrac{N}{n}$
from 2 years to 3 years	$\gamma = [(2) - (3)]\,\dfrac{N}{n^2}$
from 3 years to 4 years	$\delta = [(3) - (4)]\,\dfrac{N}{n^3}$

etc.

and from that we will find the fractions (1), (2), (3) etc. which comprises the law of mortality,

$$(1) = 1 - \frac{\alpha}{N}$$

$$(2) = (1) - \frac{n\beta}{N} = 1 - \frac{\alpha + n\beta}{N}$$

$$(3) = (2) - \frac{n^2\gamma}{N} = 1 - \frac{\alpha + n\beta + n^2\gamma}{N}$$

$$(4) = (3) - \frac{n^3\delta}{N} = 1 - \frac{\alpha + n\beta + n^2\gamma + n^3\delta}{N}$$

etc.

27. *Voilà,* a manner more certain than that of the life annuities for determining the law of mortality, and this determination will become easier if we choose a town or province where the number of burials equals the that of baptisms, so that $n = 1$. Then indeed it suffices to know the number of deaths at each age. But it must be noted that such a law of mortality need only extend to the town or province from which it was drawn. Other regions can have in place a totally different such law; and it has been observed, in particular, that in the large towns the mortality is bigger than in the small, and in the latter bigger than in the villages. If we trouble ourselves to establish well both the law of mortality and that of fertility for several places, we will be able to draw from them a number of very important conclusions.

28. But it is still necessary to note that, in the calculation that I have just developed, I have supposed that the number of all the living from a place stays the same, or increases or decreases uniformly, so that it is necessary to exclude both such extraordinary ravages as plague, war, and famine, as well as extraordinary growth, as from new colonies. It will also be good to choose such a place where all those born stay in the region and where outsiders do not come to live there and die, which would contradict the principles on which I have based the preceding calculations. In places subject to such irregularities, it would be necessary to keep exact records both of the living and the dead, and then by following the principles that I just established, one would be in a position to apply the same calculations. Everything always comes back to these two principles, that of mortality and that of fertility, which being once well established for a certain place, it is not difficult to resolve all the questions that one can propose on this matter, of which I am happy do have related the principles.

29. I have also treated these questions only in general, without limiting them to any particular place; now, to draw from them all the benefits, all depends on a large number of observations made in several different places, both of the number of all the living and of the births during one or several years, as well as of the number of deaths with their ages. Since this is something that is difficult to do well, we are to be very much indebted to Mr. Sussmilch, *Conseiller du Consistoire Supérieur*, who, after having overcome near insurmountable obstacles, came to furnish us so large a number of such observations, which appear sufficient to decide most of the questions which present themselves in this research. And indeed, he has himself drawn so many important conclusions, that we can hope that he will carry by his efforts this science to the highest degree of perfection to which it is capable.