Extract of a Letter from Mr. Euler to Mr. Beguelin, in May 1778.

I heard with pleasure the reading of the memoir of Mr. Beguelin on prime numbers, inserted into the latest volume of the *Mémoires de L'Académie Royale de Berlin*, and as I have worked for some time on the same subject, I believe that he will receive, with as much satisfaction, some observations which I have had occasion to make relative to the problem he treated in the above-mentioned memoir.

These investigations are based on this beautiful property, that all numbers which are contained in only one way in the formula $xx + yy$ are either primes or double primes, when taking the numbers x and y to be mutually prime. Now, I have noticed that several other similar formulas of the form $nxx + yy$ are endowed with the same property, and that, provided we give values to the letter n which are *suitable*, such as, for example

$$2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 12,\ 13,\ \text{etc.}$$

we always derive prime numbers from them; or indeed, that by excluding the following values of n, namely

$$11,\ 14,\ 17,\ 19,\ 20,\ 23,\ 26,\ 27,\ \text{etc.}$$

the formula $nxx + yy$ always gives prime numbers. For the number 15, for example, although contained in only one way in the formula $11xx+yy$, is a composite number. It is the same with the other numbers I just excluded, whereas those which I called suitable values reliably give up as prime every number which is contained in only one way in the form $nxx+yy$. It is therefore of utmost importance to distinguish well the suitable values of the letter n from those we must exclude in these investigations.

For this purpose, I have found and proved this rule: that if all the numbers contained in the form $n + yy$ and less than $4n$ (while taking for y numbers relatively prime to n) are either primes p, or doubles of primes $2p$, or squares of primes pp, or else finally some power of 2, then the value of n, which satisfies these conditions, may be admitted as suitable for the examination of whatever number one might propose. In this way, for example, I have found that the number 60 is in the series of suitable values, for we have

$$60 + 1^2 = 61 = p,$$

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$$60 + 7^2 = 109 = p,$$

$$60 + 11^2 = 181 = p,$$

$$60 + 13^2 = 229 = p,$$

where it is necessary to stop, since what follows surpasses the limit $4 \cdot 60$. It is the same with the number 15, since

$$15 + 1^2 = 16 = 2^4$$

$$15 + 4^2 = 31 = p.$$

By means of this rule, I was in a position to quite easily find all the values we can give to the letter n, in order that every number contained in a single way in the form $nxx + yy$ may be supposed prime. Here are these values:

1	16	48	120	312
2	18	57	130	330
3	21	58	133	345
4	22	60	165	357
5	24	70	168	385
6	25	72	177	408
7	28	78	190	462
8	30	85	210	520
9	33	88	232	760
10	37	93	240	840
12	40	102	253	1320
13	42	105	273	1365
15	44	112	280	1848.

These numbers, which, far from being scattered randomly, have a law of progression (which is clear enough when we glance through all the successive exclusions it is necessary to skip over in order to find the suitable values) and seem like they must go to infinity. So I was quite surprised to find myself finally halted at 1848, beyond which I no longer found any but unsuitable values. However, by means of the last value 1848, we are in a position to discover extremely large prime numbers, considering that nothing is easier than to examine whether a given number is contained a single time in the form $1848xx + yy$, or not, and in the former case we may boldly pronounce that this number is prime. By means of this form I have found the following primes, among others:

1016401, 1103257, 1288057, 1487641, 1702009,

2995609, 4658809, 9094009, 11866009, 18518809.

In the latter case, where the given number is contained in more than one way in the form $1848xx + yy$, it would be superfluous to note that one could very easily specify the divisors of this number.

But I think it appropriate to add that in the table of prime numbers inserted in volume XIX of the *Commentaires* of our Academy, an error slipped in because we overlooked the divisor 293, which affects only the number 1,000,009, which must be erased from this list, since it equals $293 \cdot 3413$.